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Mixed convection wall plumes

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Abstract—A proper mixed convection parameter $\zeta = (\sigma\ Ra)^{1/5}/(\omega\ Re)^{1/2}$, with $\sigma = Pr/(1+Pr)$ and $\omega = Pr/(1+Pr)^{1/3}$, is proposed to replace the conventional parameters, $Gr/Re^{5/2}$ and $Re/Gr^{2/5}$, for the analysis of mixed convection wall plumes. New coordinates, $\xi = \zeta/(1+\zeta)$ and $\eta = (y/x)[(\omega\ Re)^{1/2}+(\sigma\ Ra)^{1/5}]$, and dimensionless stream function and temperature of proper scales are also introduced. The resulting non-similar equations are solved by using a very effective finite-difference scheme. The obtained solutions are uniformly valid over the entire regime of mixed convection intensity from forced convection limit to free convection limit for fluids of any Prandtl number between 0.001 and 1000.

INTRODUCTION

MIXED convection wall plumes, which arise from a line thermal source embedded at the leading edge of a vertical flat plate, have only been studied recently by Rao *et al.* [1], and by Krishnamurthy and Gebhart [2]. However, the previous local similarity and non-similarity solutions [1] and the matched asymptotic expansions solution [2] do not cover the real mixed convection regime where the buoyancy force and the inertia force are comparable in magnitude. In addition, these solutions are presented only for Prandtl numbers around 1, i.e. $Pr = 0.7$ and 6.7 or 7. Mixed convection wall plumes in fluids of very small and very large Prandtl numbers have not been investigated previously.

In the present analysis of mixed convection wall plumes, we propose a proper mixed convection parameter $\zeta = (\sigma\,Ra)^{1/5}/(\omega\,Re)^{1/2}$, with $\sigma = Pr/(1+Pr)$ and $\omega = Pr/(1+Pr)^{1/3}$, to replace the conventional mixed convection parameters Gr^2/Re^5 [3], $Gr/Re^{5/2}$ and $Re/Gr^{2/5}$ [1]. This dimensionless group serves as a controlling parameter that determines the relative importance of the forced and the free convection for fluids of any Prandtl number. For large values of Prandtl number, ζ reduces to $Ra^{1/5}/Re^{1/2}\,Pr^{1/3}$, while for small values of Prandtl number, $\zeta = (Pr\,Ra)^{1/5}/(Pr\,Re)^{1/2}$. New coordinates, $\xi = \zeta/(1+\zeta)$ and $\eta = (y/x)\lambda$ where $\lambda = [(\omega\,Re)^{1/2}+(\sigma\,Ra)^{1/5}] = (\omega\,Re)^{1/2}(1+\zeta)$, are defined based on ζ. In addition, dimensionless stream function and dimensionless temperature with proper scales are defined based on λ. The similarity variable $\eta = (y/x)(\omega\,Re)^{1/2}$ has recently been proposed in ref. [4] for the forced convection heat transfer from wedges to fluids of any Prandtl number ($0.0001 \leqslant Pr \leqslant \infty$). The corresponding similarity variable $\eta = (y/x)(\sigma\,Ra/4)^{1/4}$ for free convection from an isothermal vertical plate were introduced by Larsen and Arpaci [5].

The dimensionless group ζ (or ξ) is a stretched streamwise coordinate measured from the leading edge. It also serves as an index of the relative strength of the buoyancy force to the inertia force in the flow. For the limiting case of pure forced convection, $\zeta = 0$ and $\xi = 0$. While for the pure free convection limit, $\zeta \to \infty$ and $\xi = 1$. The two physical interpretations of ζ (or ξ) are consistent. At the neighbourhood near the leading edge, forced convection is dominant in the near-field flow, whereas free convection is dominant in the far-field flow in the downstream region.

The new formulation derived for mixed convection wall plumes was solved by a very effective numerical scheme which was developed in ref. [6] for the analyses of the plumes with governing equations of non-similar type. Numerical results are uniformly valid over the entire range of mixed convection intensity from the pure forced convection limit ($\xi = 0$) to the pure free convection limit ($\xi = 1$) for fluids of any Prandtl number from 0.001 to 1000.

ANALYSIS

We consider an incompressible, laminar stream flowing parallel to a semi-infinite adiabatic vertical plate. A line thermal source, which constantly releases heat at a rate of Q per unit length, is embedded at the leading edge of the plate. Buoyancy-assisted plumes or buoyancy-opposed plumes would arise depending on the forced stream flows upward or downward, respectively. The boundary-layer equations for the mixed convection wall plumes, using the Boussinesq approximation, are given by

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y} = 0 \tag{1}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y} = \nu\frac{\partial^2 u}{\partial y^2}+\varepsilon g\beta(T-T_\infty) \tag{2}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y} = \alpha\frac{\partial^2 T}{\partial y^2} \tag{3}$$

where ε is an index. For buoyancy-assisted plumes, $\varepsilon = 1$, whereas for buoyancy-opposed plumes, $\varepsilon = -1$. The boundary conditions are

$$u = 0,\quad v = 0,\quad \partial T/\partial y = 0 \qquad \text{at } y = 0 \tag{4}$$

$$u = u_\infty,\quad T = T_\infty \qquad \text{as } y \to \infty. \tag{5}$$

In addition, the conservation of energy requires that

NOMENCLATURE

C_f local friction coefficient, $2\tau_w/\rho u_\infty^2$
C_p specific heat capacity
f reduced stream function
g gravitational acceleration
L length of the line thermal source
Pr Prandtl number, ν/α
Q rate of heat released by the line source per unit length
Ra local Rayleigh number, $g\beta T^* x^3/\alpha\nu$
Re local Reynolds number, $u_\infty x/\nu$
T fluid temperature
T_w wall temperature
T_∞ free stream temperature
T^* characteristic temperature of the line source, $Q/(\rho C_p \alpha L)$
u velocity component of the x-coordinate
u_∞ free-stream velocity
v velocity component in the y-direction
x coordinate from leading edge
y coordinate normal to plate.

Greek symbols
α thermal diffusivity
β thermal expansion coefficient
ζ mixed convection parameter, $(\sigma Ra)^{1/5}/(\omega Re)^{1/2}$
η pseudo-similarity variable, $(y/x)\lambda$
θ dimensionless temperature, $[(T-T_\infty)/T^*]\lambda$
λ $(\omega Re)^{1/2}+(\sigma Ra)^{1/5}$
μ dynamic viscosity
ν kinematic viscosity
ξ $\zeta/(1+\zeta)$
ρ density
σ $Pr/(1+Pr)$
τ_w wall shear stress
ψ stream function
ω $Pr/(1+Pr)^{1/3}$.

the convective energy carried by the boundary-layer flow, across the horizontal plane at any $x > 0$, is equal to the heat released by the line source. Hence

$$Q = \rho C_p L \int_0^\infty u(T-T_\infty)\,dy. \tag{6}$$

In the present analysis, we propose

$$\zeta = (\sigma Ra)^{1/5}/(\omega Re)^{1/2} \tag{7}$$

to replace the conventional mixed convection parameters Gr^2/Re^5 [3], $Gr/Re^{5/2}$ and $Re/Gr^{2/5}$ [1], where $Re = u_\infty x/\nu$ and $Ra = g\beta T^* x^3/\alpha\nu$ are the local Reynolds and Rayleigh numbers, respectively, and $\sigma = Pr/(1+Pr)$, $\omega = Pr/(1+Pr)^{1/3}$. The characteristic temperature of the line source is defined as

$$T^* = Q/(\rho C_p \alpha L). \tag{8}$$

Furthermore, we introduce new coordinates

$$\xi(x) = \zeta/(1+\zeta) \quad \text{and} \quad \eta(x,y) = (y/x)\lambda \tag{9}$$

where the unified mixed-flow parameter λ is defined by

$$\begin{aligned}\lambda &= [(\omega Re)^{1/2}+(\sigma Ra)^{1/5}] \\ &= (\omega Re)^{1/2}/(1-\xi) = (\sigma Ra)^{1/5}/\xi.\end{aligned} \tag{10}$$

In terms of λ and T^*, a reduced stream function $f(\xi,\eta)$ and a dimensionless temperature $\theta(\xi,\eta)$ are respectively defined as

$$f(\xi,\eta) = \psi/\alpha\lambda \quad \text{and}$$
$$\theta(\xi,\eta) = [(T-T_\infty)/T^*]\lambda \tag{11}$$

where the stream function $\psi(x,y)$ satisfies the continuity equation (1), with $u = \partial\psi/\partial y$ and $v = -\partial\psi/\partial x$.

By substituting equations (7)–(11) into equations (1)–(6), we obtain

$$Pr\,f''' + \frac{5+\xi}{10}ff'' - \frac{\xi}{5}f'f' + \varepsilon(1+Pr)\xi^5\theta = \frac{1}{10}\xi(1-\xi)\left[f'\frac{\partial f'}{\partial \xi} - f''\frac{\partial f}{\partial \xi}\right] \tag{12}$$

$$\theta'' + \frac{5+\xi}{10}(f\theta)' = \frac{1}{10}\xi(1-\xi)\left[f'\frac{\partial \theta}{\partial \xi} - \theta'\frac{\partial f}{\partial \xi}\right] \tag{13}$$

$$f(\xi,0) = 0, \quad f'(\xi,0) = 0, \quad \theta'(\xi,0) = 0 \tag{14}$$

$$f'(\xi,\infty) = (1-\xi)^2(1+Pr)^{1/3}, \quad \theta(\xi,\infty) = 0 \tag{15}$$

$$\int_0^\infty f'\theta\,d\eta = 1. \tag{16}$$

Equations (12) and (13) are readily reduced to a set of self-similar equations for the special cases of a pure forced convection plume ($\xi = 0$) and a pure free convection plume ($\xi = 1$), separately.

RESULTS AND DISCUSSION

The set of non-similarity equations (12) and (13) subject to boundary conditions (14) and (15) and the integral equation (16) is solved by a very effective finite-difference scheme developed recently in ref. [6]. This numerical scheme is a modified version of Keller's box method [7]. The essential modification is that an additional iteration scheme was joined to Keller's procedure to deal with the integral equation (16) of normalized flux conservation. Details of the numerical method can be found in refs. [6, 7]. Numerical results of $f''(\xi,0)$ and $\theta(\xi,0)$ for buoyancy-assisted plumes are presented in Tables 1 and 2, respectively.

The evolution of the profiles of the dimensionless velocity $u/(\alpha/x)\lambda^2 = f'(\xi,\eta)$ from the pure forced convection limit ($\xi = 0$) to the pure free convection limit ($\xi = 1$) are explicitly illustrated in Fig. 1 for

Table 1. Results of $f''(\xi, 0)$ for buoyancy-assisted plumes

ξ	Pr 0.001	0.01	0.1	0.7	7	100
0	10.500	3.3370	1.1013	0.51748	0.35518	0.33371
0.1	7.6587	2.4335	0.80306	0.37732	0.25934	0.24333
0.2	5.4027	1.7163	0.56626	0.26604	0.18384	0.17174
0.3	3.8361	1.2167	0.40017	0.18785	0.13225	0.12285
0.4	3.4861	1.0964	0.35448	0.16509	0.11955	0.11335
0.5	5.2880	1.6370	0.51107	0.23038	0.16333	0.15962
0.6	9.7391	2.9681	0.89591	0.38650	0.26303	0.25680
0.7	16.729	5.0421	1.4903	0.62408	0.41341	0.40093
0.8	26.088	7.8182	2.2873	0.94340	0.61621	0.59538
0.9	37.837	11.316	3.2969	1.3512	0.87720	0.84620
1.0	52.114	15.585	4.5362	1.8562	1.2033	1.1602

Table 2. Results of $\theta(\xi, 0)$ for buoyancy-assisted plumes

ξ	Pr 0.001	0.01	0.1	0.7	7	100
0	0.56429	0.56511	0.57102	0.58783	0.60594	0.60960
0.1	0.62693	0.62786	0.63444	0.65311	0.67297	0.67737
0.2	0.70522	0.70626	0.71364	0.73459	0.75584	0.76176
0.3	0.80467	0.80579	0.81392	0.83700	0.85652	0.86514
0.4	0.92656	0.92750	0.93452	0.95454	0.95960	0.96571
0.5	1.0402	1.0403	1.0407	1.0408	1.0126	0.99450
0.6	1.0808	1.0798	1.0722	1.0443	0.98184	0.93463
0.7	1.0419	1.0404	1.0282	0.98316	0.90031	0.83999
0.8	0.96388	0.96233	0.94883	0.89831	0.80882	0.74765
0.9	0.87664	0.87518	0.86205	0.81265	0.72518	0.66820
1.0	0.79334	0.79202	0.77996	0.73450	0.65331	0.60188

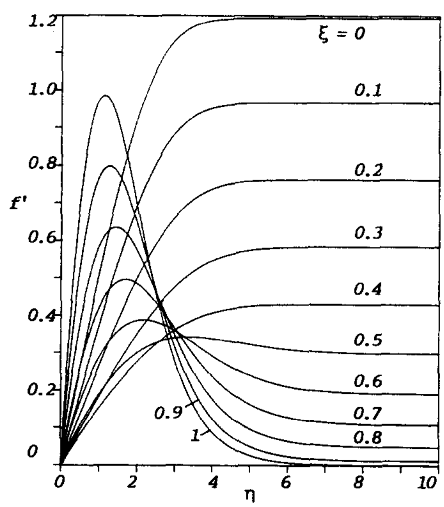

FIG. 1. Profiles of $f'(\xi, \eta)$ for buoyancy-assisted plumes, $Pr = 0.7$.

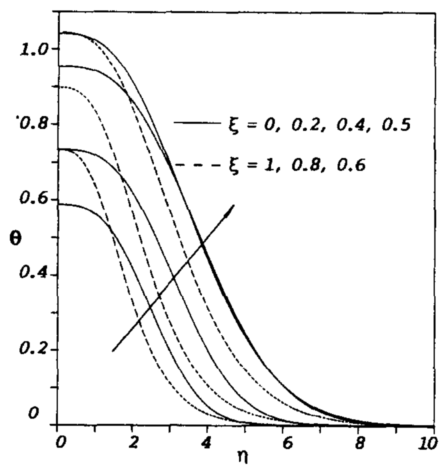

FIG. 2. Profiles of $\theta(\xi, \eta)$ for buoyancy-assisted plumes, $Pr = 0.7$.

buoyancy-assisted plumes. The evolution of the dimensionless temperature profiles $\theta(\xi, \eta)$ is shown in Fig. 2. These figures also represent the variations of $f'(\xi, \eta)$ and $\theta(\xi, \eta)$ with increasing streamwise distance from the line source.

The wall shear stress

$$\tau_w = \mu(\partial u/\partial y)_{y=0} = \rho(\alpha v/x^2)\lambda^3 f''(\xi, 0) \quad (17)$$

is usually expressed in terms of the local friction coefficient $C_f = 2\tau_w/(\rho u_\infty^2)$ as

$$C_f Re^{1/2} = 2[\sigma^{1/2}/(1-\xi)^3] f''(\xi, 0). \quad (18)$$

It is seen from Fig. 3 that $C_f Re^{1/2}$ initially equals the forced convection value 0.66411 and increases with increasing mixed convection parameter ζ for the buoyancy-assisted flow condition. On the other hand, $C_f Re^{1/2}$ decreases as ζ increases for the buoyancy-opposed flow condition. A very sharp decrease in wall friction can be seen in Fig. 3 when the critical value of ζ is approached. This indicates a breakdown of the boundary-layer approximation, as recently noticed by Schneider and Wasel [8] from mixed convection above a horizontal plate. The critical values of ζ, beyond which the finite-difference solution diverges and $d(C_f Re^{1/2})/d\zeta \gg 1$, are slightly larger than 0.667 for $Pr = 0.1$, 0.7 and 7. The reported critical values of $Gr/Re^{5/2}$, at which flow separation occurs, are 0.1534 and 0.0601 [1] for $Pr = 0.7$ and 7, respectively. These values are converted to $\zeta = 0.6516$ for $Pr = 0.7$ and $\zeta = 0.6459$ for $Pr = 7$ by using the conversion relation $\zeta = \omega^{0.1}(Gr/Re^{5/2})^{1/5}$.

The influence of mixed convection intensity ζ on the dimensionless wall temperature $[(T_w - T_\infty)/T^*](\omega\, Re)^{1/2}$ of buoyancy-assisted plumes is presented in Fig. 4 for fluids of $Pr = 0.001$–1000. It can be seen from Fig. 4 that the dimensionless wall temperature $[(T_w - T_\infty)/T^*](\omega\, Re)^{1/2}$ decreases as the mixed convection parameter ζ increases. Further examination of Figs. 3 and 4 reveals that the knees of all the curves are at $\zeta = O(1)$ for all Prandtl numbers. Consequently, the present mixed convection parameter ζ is the dimensionless group that properly serves as the controlling parameter governing the relative importance of the forced and the free convection for fluids of any Prandtl number.

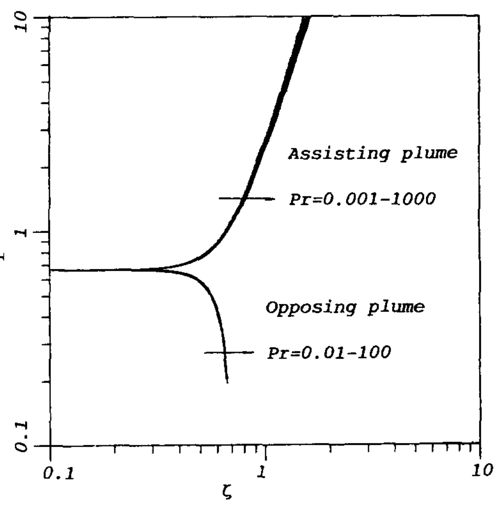

FIG. 3. Variation of $C_f Re^{1/2}$ with ζ.

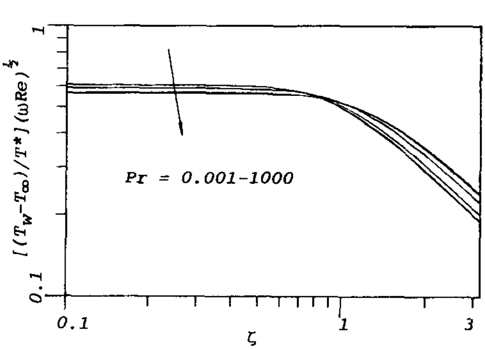

FIG. 4. Variation of $[(T_w - T_\infty)/T^*](\omega\, Re)^{1/2}$ with ζ.

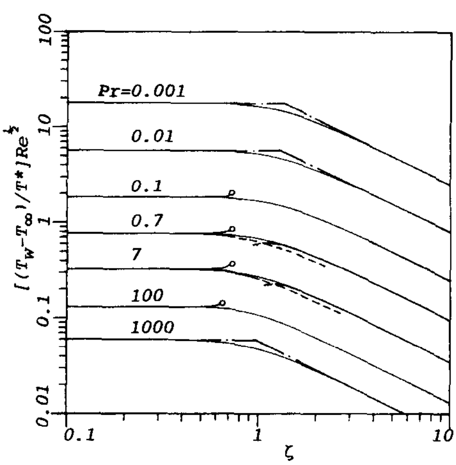

FIG. 5. Variation of $[(T_w - T_\infty)/T^*]Re^{1/2}$ with ζ: - - - -, local non-similarity solutions [1]; ——○, breakdown point for buoyancy-opposed plumes; —·—, asymptotes of convection limits.

Figure 4 was replotted as Fig. 5 with $[(T_w - T_\infty)/T^*]Re^{1/2}$ as the ordinate to separate the curves for various Prandtl numbers. The dimensionless wall temperature $[(T_w - T_\infty)/T^*]Re^{1/2}$ for the buoyancy-opposed plume is also presented in Fig. 5. It increases with increasing ζ until the point of breakdown is approached. Asymptotes of the convection limits are shown in this figure for $Pr = 0.001$, 0.01 and 1000.

The previous local similarity and local non-similarity solutions [1] of weakly buoyant plumes and strongly buoyant plumes for $Pr = 0.7$ and 7 are also presented in Fig. 5 for comparison. As expected, these approximate solutions coincide with the present solutions in the regions near the forced convection limit and the free convection limit where they are valid.

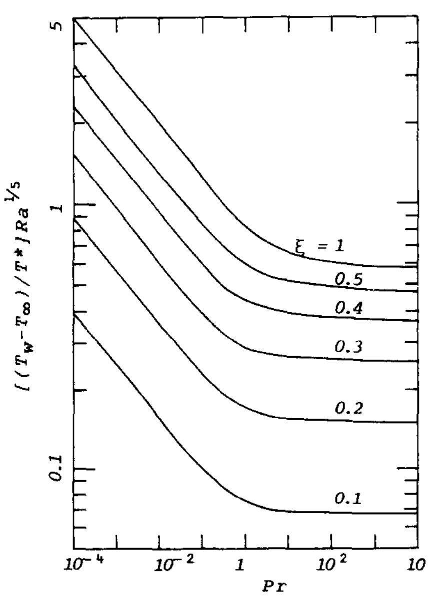

FIG. 6. Variation of $[(T_w - T_\infty)/T^*]Ra^{1/5}$ with Pr.

However, the local non-similarity solutions deviate from the finite-difference solutions when the strength of the inertia force and buoyancy force are comparable in magnitude. At the real mixed convection regime, the accuracy of the previous approximate solutions deteriorates [1, 2].

The variation of the dimensionless wall temperature $[(T_w - T_\infty)/T^*]Ra^{1/5}$ as a function of the Prandtl number is presented in Fig. 6 for some specified values of ξ. From Fig. 6, it can be seen that this dimensionless wall temperature decreases linearly from very small values of Prandtl number to $Pr < 0.1$ and that the rate of decrease slows down for $Pr > 0.1$. These curves approach the asymptotes of $Pr \to \infty$ when $Pr > 100$.

The accuracy of the present finite-difference solutions can also be verified by comparing the calculated dimensionless wall temperature $[(T_w - T_\infty)/T^*]Ra^{1/5}$ and the wall friction $\tau_w/[\rho(\alpha\nu/x^2)Ra^{3/5}]$ for the special case of the free convection limit ($\xi = 1$) with the reported data of the free convection wall plume [9].

Table 3. Comparison of wall temperature $[(T_w - T_\infty)/T^*]Ra^{1/5}$ and wall friction $\tau_w/\rho(\alpha\nu/x^2)Ra^{3/5}$ for the pure free convection wall plume ($\xi = 1$)

	$[(T_w - T_\infty)/T^*]Ra^{1/5}$		$\tau_w/\rho(\alpha\nu/x^2)Ra^{3/5}$	
Pr	Present	Liburdy and Faeth	Present	Liburdy and Faeth
0.001	3.1590	—	0.82103	—
0.01	1.9934	1.9936	0.97751	0.97789
0.1	1.2599	1.2601	1.0761	1.0763
0.7	0.87712	0.87713	1.0900	1.0900
1	0.82874	0.83397	1.0899	1.0950
7	0.67099	—	1.1107	—
10	0.65462	0.65471	1.1174	0.87962
100	0.60308	0.60381	1.1533	1.1542
1000	0.58782	—	1.1702	—

Excellent agreement between these two solutions can be seen in Table 3. Since the set of equations are integrated step by step from $\xi = 0$ ($x = 0$) to $\xi = 1$, the accurate results of $\xi = 1$ ensure that the finite-difference solutions are uniformly valid over the entire range of the mixed convection regime from the forced convection limit to the free convection limit.

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CONVECTION MIXTE DANS LES PANACHES PARIETAUX

Résumé—On propose un paramètre approprié à la convection mixte $\zeta = (\sigma\ Ra)^{1/5}/(\omega\ Re)^{1/2}$, avec $\sigma = Pr/(1+Pr)$ et $\omega = Pr/(1+Pr)^{1/3}$ pour remplacer les paramètres conventionnels, $Gr/Re^{5/2}$ et $Re/Gr^{2/5}$, dans l'analyse de la convection mixte pour les panaches pariétaux. De nouvelles coordonnées, $\xi = \zeta/(1+\zeta)$ et $\eta = (y/x)\ [(\omega\ Re)^{1/2}+(\sigma\ Ra)^{1/5}]$, une fonction de courant et une temperature adimensionnelles d'échelles convenables sont introduites. Les équations résultantes non similaires sont résolues en utilisant un schéma aux différences finies trés performant. Les solutions obtenues sont uniformément valables dans la région entière de la convection mixte, depuis la convection forcée jusqu'à la convection naturelle, pour les fluides à nombre de Prandtl quelconque entre 0,001 et 1000.

WANDNAHE AUFTRIEBSFAHNEN BEI MISCHKONVEKTION

Zusammenfassung—Als Ersatz für die herkömmlichen Parameter $Gr/Re^{5/2}$ und $Re/Gr^{2/5}$ zur Analyse der wandnahen Auftriebsfahnen infolge Mischkonvektion wird ein Parameter $\zeta = (\sigma\ Ra)^{1/5}/(\omega\ Re)^{1/2}$ vorgeschlagen, mit $\sigma = Pr/(1+Pr)$ und $\omega = Pr/(1+Pr)^{1/3}$. Außerdem werden neue Koordinaten, $\xi = \zeta/(1+\zeta)$ und $\eta = (y/x)\ [(\omega\ Re)^{1/2}+(\sigma\ Ra)^{1/5}]$ sowie eine dimensionslose Stromfunktion und Temperatur mit geeigneter Normierung eingeführt. Die sich ergebenden Nicht-Ähnlichkeits-Gleichungen werden unter Benutzung eines hocheffizienten Finite-Differenzen-Verfahrens gelöst. Die gewonnenen Lösungen gelten einheitlich über den gesamten Bereich der Mischkonvektion von der Grenze für erzwungene Konvektion bis zur Grenze für freie Konvektion für Fluide mit Prandtl-Zahlen zwischen 0,001 und 1000.

СМЕШАННОКОНВЕКТИВНОЕ ПОДЪЕМНОЕ ПРИСТЕННОЕ ТЕЧЕНИЕ

Аннотация—При анализе пристенной подъемной смешанной конвекции предлагается использовать вместо общепринятых параметров $Gr/Re^{5/2}$ и $Re/Gr^{2/5}$ характерный параметр смешанной конвекции $\zeta = (\sigma\, Ra)^{1/5}/(\omega\, Re)^{1/2}$, где $\sigma = Pr/(1 + Pr)$ и $\omega = Pr/(1 + Pr)^{1/3}$. Вводятся новые координаты $\xi = \zeta/(1 + \zeta)$ и $\eta = (y/x)[(\omega\, Re)^{1/2} + (\sigma\, Ra)^{1/5}]$, а также с помощью характерных масштабов—безразмерные функция тока и температура. С помощью эффектиных конечно-разностных схем решены результирующие неавтомодельные уравнения. Полученные решения одинаково справедливы во всем диапазоне смешанной конвекции от предельного случая вынужденной конвекции до чисто естественной для жидкостей с числами Прандтля от 0,001 до 1000.